

Single- j -shell studies of cross-conjugate nuclei and isomerism

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Isomeric states for 4 nucleons with isospin $T = 1$ are here considered. A comparison is made of the lighter and heavier members of cross-conjugate pairs. Although in the single j shell the spectra in the two cases should be identical, this is not the case experimentally. For the former, the ground states all have angular momentum $J = 2$. This result is found in a single- j -shell calculation when the interaction is obtained from the spectrum of two particles. In a single j shell ($f_{7/2}$, $g_{9/2}$, $h_{11/2}$), the state with median angular momentum $J = (J_{\max} + 1)/2$ is the ground state for the heavier member of the pair provided one uses as an interaction the spectrum of two holes. The ground state behaviour can also be explained by rotational models. A new observation is that both in single- j and in experiment the $J = 2$ state in the heavier member and the $J = (J_{\max} + 1)/2$ state in the lighter member are isomeric. This criss-cross behaviour shows that some remnants of the single- j -shell model persists and indeed this model works surprisingly well.

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I. INTRODUCTION

In this work we develop a rule based on interesting behaviours of nuclear spectra, or to be more precise spectra of four-nucleon states with isospin $T = 1$ in odd-odd nuclei. Such states consist of either three protons and one neutron or three neutrons and one proton; also three proton holes and one neutron hole or three neutron holes and one proton hole. We find in single- j -shell calculations that in the $f_{7/2}$, $g_{9/2}$, and $h_{11/2}$ shells, states with median total angular momentum $J = (J_{\max} + 1)/2$ lie low in energy and become isomeric for lighter members of cross-conjugate pairs and ground states for the heavier members. Conversely, $J = 2^+$ states are ground states for the lighter members and isomeric for the heavier members. Although these calculations are relatively simple—not large scale—they are supported by experiment. We note that J_{\max} is equal to M_{\max} . For three neutrons the maximum value of M is $j + (j - 1) + (j - 2)$ and for the single proton it is j . Thus J_{\max} is equal to $(4j - 3)$ whilst $(J_{\max} + 1)/2$ is equal to $(2j - 1)$. To briefly summarise the findings, we note that for the three shells listed above the values of J_{\max} are 11, 15, and 19, respectively. Thus the $(J_{\max} + 1)/2$ rule gives values of 6, 8, and 10 for the low-lying isomeric (or ground) states. We emphasize that the single- j -shell model is used only to make qualitative statements about isomerism.

II. THE $f_{7/2}$ SHELL

We start with the $f_{7/2}$ shell where single- j -shell calculations have already been performed and wave functions tabulated by Zamick, Escuderos, and Bayman [1]; this reference is based on previous work of Refs. [2–4]. The interaction used consists of matrix elements taken

from experiment—more precisely from the spectrum of ^{42}Sc and ^{42}Ca (INTa). Zamick, however, noted that for the upper half of the $f_{7/2}$ shell one obtains better results by using matrix elements from the two-hole system ^{54}Co (INTb) [5]. In single- j -shell calculations with both neutrons and protons, we define the cross-conjugate of a given nucleus as one in which protons are replaced by neutron holes and neutrons by proton holes. Thus ^{52}Fe is the cross-conjugate of ^{44}Ti and ^{52}Mn is the cross-conjugate of ^{44}Sc . If one uses the same charge independent two-body interaction in both nuclei, the spectra for states in this limited model space should be identical. In fact, although the spectra are similar, they are not identical experimentally. The 10^+ state in ^{44}Ti is below the 12^+ , but in ^{52}Fe the reverse is true. In both cases the 12^+ state is isomeric but the one in ^{52}Fe has a much longer half-life because it cannot decay to the 10^+ state. As seen in Table I we are successful in getting the 12^+ below the 10^+ by using the spectrum of ^{54}Co as input. The main difference in the two-body spectra is that the $J = 7^+$ state in ^{54}Co is much lower in energy than it is in ^{42}Sc (see Table VII).

Large space shell-model calculations for ^{52}Fe were performed by Ur *et al.* [6] using the KB3 interaction and by Puddu [7] using the GXPF1A interaction. Both groups get a near degeneracy of 10_1^+ and 12_1^+ in ^{52}Fe . Thus, although they do not get 12^+ sufficiently below 10^+ , they do go on the right direction relative to ^{44}Ti . Ur *et al.* attribute increased collectivity in ^{52}Fe mainly to $p_{3/2}$ admixtures for the reason there are differences in the cross-conjugate pairs.

We then examine the yrast spectrum of ^{44}Sc calculated with the interaction INTa (see Table II). We consider two groups. First for $J = 6, 5, 4, 3, 2$, and 1, the energies in MeV are respectively 0.38, 1.28, 0.71, 0.76, 0.00, and 0.43 (the $J = 0^+$ state has isospin $T = 2$ and is at an excitation energy of 3.047 MeV). We see that the only state below the $J = 6$ state is $J = 2$. Thus, the lowest multipolarity for decay is $E4$ and so the $J = 6$ state is calculated to be isomeric. For the second group with $J =$

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Table I: Yrast spectra of ^{44}Ti and ^{52}Fe calculated with the interactions INTa and INTb respectively (see text) and compared with experiment [8].

$E(\text{MeV})$				
^{44}Ti			^{52}Fe	
J	INTa	Exp.	INTb	Exp.
0	0.000	0.000	0.000	0.000
1	5.669		5.442	
2	1.163	1.083	1.015	0.849
3	5.786		5.834	
4	2.790	2.454	2.628	2.384
5	5.871		6.463	
6	4.062	4.015	4.078	4.325
7	6.043		5.890	
8	6.084	(6.509)	5.772	6.361
9	7.984		7.791	
10	7.384	(7.671)	6.721	7.382
11	9.865		8.666	
12	7.702	(8.040)	6.514	6.958

11, 10, 9, 8, and 7, the energies in MeV are respectively 4.64, 4.79, 3.39, 3.10, and 1.27. The $J^\pi = 11^+$ state can decay via an $E2$ transition to the $J^\pi = 9^+$ state so it should not be isomeric.

Table II: Yrast spectra of ^{44}Sc and ^{52}Mn calculated with the interactions INTa and INTb respectively (see text) and compared with experiment [8].

$E(\text{MeV})$				
^{44}Sc			^{52}Mn	
J	INTa	Exp.	INTb	Exp.
0	3.047		2.774	
1	0.432	0.667	0.443	0.546
2	0.000	0.000	0.202	0.378
3	0.764	0.762	0.836	0.825
4	0.713	0.350	0.851	0.732
5	1.276	1.513	1.404	1.254
6	0.381	0.271	0.000	0.000
7	1.272	0.968	1.819	0.870
8	3.097		2.572	(2.286)
9	3.390	2.672	2.792	(2.908)
10	4.793	4.114	4.365	4.164
11	4.638	3.567	3.667	(3.837)

We now look at experiment. In ^{44}Sc the lowest $J^\pi = 6^+$ state has a half-life of 58.6 hours—it is indeed isomeric.

But we should also consider the cross-conjugate nucleus ^{52}Mn consisting of three proton holes and one neutron hole relative to ^{56}Ni . We see that here the $J^\pi = 6^+$ state is the ground state with a half-life of 5.591 days. As mentioned before, if we use the same interaction here as we did for ^{44}Sc , we would not get the $J = 6^+$ state as the ground state. But as seen in Table II, when we use as input the spectrum of the two-hole system ^{54}Co , we do get $J = 6^+$ as the ground state.

There is some indication that in heavier nuclei the state with $J = J_{\text{max}}$ should be isomeric. However, the $J^\pi = 11^+$

state at 3.57 MeV in ^{44}Sc has a half-life of 48 ps whilst the corresponding $J^\pi = 11^+$ state in ^{52}Mn at 3.84 MeV has a half-life of 15.1 ps.

We could not find large space shell-model calculations of ^{52}Mn in the literature, but there is a single- j -shell calculation in the work of Avrigeanu *et al.* [9]. This accompanies their experimental work on high-spin states in this nucleus.

III. THE $g_{9/2}$ SHELL

In previous work [10], calculations were performed in the $g_{9/2}$ shell where the emphasis was on partial dynamical symmetries. However, many residual calculations were lying around which had not been carefully examined. Nara Singh [11] reported the finding by his group of a $J^\pi = 16^+$ isomeric state in ^{96}Cd that beta decayed to a $J^\pi = 15^+$ state in ^{96}Ag , which is also isomeric. This largely stimulated the work done here on isomerism. We also note a combination of experiment and shell-model calculations by K. Schmidt *et al.* [12] and L. Batist *et al.* [13]. The topics addressed in these works are decay properties of very neutron-deficient isotopes of silver and cadmium, as well as isomerism in ^{96}Ag .

We show results for two interactions: INTc and INTd. The $T = 1$ matrix elements are obtained from the spectrum of ^{98}Cd , that is, two proton holes. Unfortunately, the spectrum of ^{98}In is not known, so we cannot get the $T = 0$ matrix elements from experiment. We use a delta interaction to generate the $T = 0$ matrix elements for INTc. Noting that in the $f_{7/2}$ shell the state with $J = J_{\text{max}}$, i.e. $J = 7$, comes much lower for two holes than it does for two particles, we simulate this behaviour in INTd in the $g_{9/2}$ shell by changing the $J_{\text{max}} = 9$ energy from 1.4964 MeV to 0.7500 MeV, leaving all other two-body matrix elements the same. This interaction should be more appropriate for the four-hole system.

With the INTc interaction, the $J = 2^+$ state is the ground state and should be long lived. The $J = 8^+$ is at an excitation energy of 0.350 MeV, so only the $J = 0^+$ ($T = 2$) and $J = 2^+$ states are below it. So this state should be isomeric. But for INTd, where we lowered the energy of the $J = 9^+$ two-body matrix element, the $J = 8^+$ state is now the ground state and is of course long lived. The $J = 2^+$ state is very low lying (0.097 MeV) and is isomeric. At high spin with INTd the $J = 15^+$ state is at 2.645 MeV while the $J = 13^+$ state is at 2.556 MeV. Because they are so close in energy, the $J = 15^+$ state is isomeric.

Concerning experiment in Refs. [12, 13], nearly degenerate $J^\pi = 2^+$ and $J^\pi = 8^+$ isomers are shown with respective half-lives of 6.9(6) s and 4.40(6) s. We see that also in this shell the $(J_{\text{max}} + 1)/2$ rule is verified.

We find that, unlike in the $f_{7/2}$ shell, here in $g_{9/2}$ our calculation with INTd leads to an isomeric state for $J = J_{\text{max}} = 15$ and this supports the experimental findings of Nara Singh [11]. We now refer to the experimental

Table III: Energy levels for the case of 3 protons and 1 neutron in the $g_{9/2}$ shell with the interactions INTc and INTd (see text), and compared with the experimental data for ^{96}Ag .

J	$E(\text{MeV})$		
	INTc	INTd	Exp.
0	0.246	0.900	
1	0.463	0.483	
2	0.000	0.097	
3	0.638	0.588	
4	0.394	0.349	
5	0.774	0.737	
6	0.450	0.371	
7	0.850	0.861	
8	0.350	0.000	0.000
9	0.872	0.492	0.470
10	2.188	1.748	(1.719)
11	2.344	1.930	(1.976)
12	3.004	2.550	
13	3.087	2.556	2.643
14	3.382	3.070	
15	3.287	2.645	2.643+x

works of Grzywacz *et al.* [14] and Grawe *et al.* [15]. The latter work also includes large-scale shell-model calculations and points out that there are many spin-gap states in the ^{100}Sn region. A near degeneracy of the two states in ^{96}Ag is shown in Fig. 1 of Grawe *et al.*, however with the $J=13^+$ state ever so slightly below the $J=15^+$ state.

IV. THE $h_{11/2}$ SHELL

We include here results for the $h_{11/2}$ shell. A $Q \cdot Q$ interaction is used with strength such that the $J=2-J=0$ splitting for two nucleons is 0.244 MeV. No comparison with experiment can be made, but we want to show that the pattern of behavior of the previous shells persists here as well. The energies in MeV for the first group with $J=10, 9, 8, 7, 6, 5, 4, 3, 2$, and 1 are respectively 0.21, 1.30, 1.58, 1.33, 1.03, 0.75, 0.49, 0.28, 0.11, and 0.00. Only states with $J=2$ and 1 lie below the $J=10$ state in this calculation. This fits in very nicely with the $(J_{\max} + 1)/2$ rule. For the second group with angular momenta $J=19, 18, 17, 16, 15, 14, 13, 12$, and 11, the energies in MeV are respectively 2.43, 3.11, 2.85, 3.08, 2.66, 2.47, 1.97, 1.46, and 0.85. The highest-spin state below the $J^\pi=19^+$ state in this calculation has $J^\pi=13^+$. Thus the $J^\pi=19^+$ state is predicted to be isomeric.

V. LIGHTER NUCLEI

The single-shell model is not expected to work for light nuclei and no calculation will be attempted. Still it is of interest to show the systematics of these nuclei in the $d_{5/2}$ and $p_{3/2}$ shells.

The nucleus ^{20}F is a special case. Here $(J_{\max} + 1)/2$

is equal to 4. Clearly this state can readily decay to the lower $J^\pi = 2^+$ state. Experimentally the lowest 3 states are: $J^\pi = 2^+$ (ground), $J^\pi = 3^+$ at 0.656 MeV, and $J^\pi = 4^+$ at 0.823 MeV. It is understandable that when $(J_{\max} + 1)/2$ differs from the ground state spin by only 2 units the isomerism is no longer present. However, despite two decay channels being open, the half-life of the 4^+ state is surprisingly long at 55 ps. The next longest half-life listed is 1.36 ps for the $J^\pi = 1^-$ state at 0.984 MeV. In the cross-conjugate nucleus ^{24}Na the $J = 4^+$ state is the ground state and is isomeric with a lifetime of 14.997 h. There is an excited 1^+ state at 0.472 MeV with a half-life of 20.18 ms and a 2^+ state at 0.563 MeV with a half-life of 36 ps. Other transitions in this nucleus are much shorter.

For ^8Li the value of $(J_{\max} + 1)/2$ is 2. This corresponds to the ground state which is of course isomeric with a half-life of 839.9 ms. This nucleus is its own cross-conjugate.

VI. EXPLANATIONS OF THE ISOMERISMS

We have admittedly done some very simple calculations, but that is the point. One should do such calculations to search for interesting behaviors. Later one can supplement these with more detailed calculations. The simple calculations are useful when effects are large as in the case of the $(J_{\max} + 1)/2$ rule.

A key to understanding the isomerisms comes from the works of Gallagher-Moszkowski [16]. They developed a scheme for obtaining and predicting the ground state spins of odd-odd nuclei. Briefly stated, the value of the total angular momentum is predicted to be $(\Omega_p + \Omega_n)$ where Ω is the component of the angular momentum along the symmetry axis. We can make a connection with this by noting that for all the heavier members of the cross-conjugate pairs in the previous sections, the ground states have J values equal to $(J_{\max} + 1)/2$, which is the same as $(\Omega_n + \Omega_p)$. In more detail the lighter members of the cross-conjugate pairs have one proton with $\Omega_p = 1/2$ and three neutrons with $\Omega_n = 3/2$. This leads to a ground state spin $J = 2$ which is verified experimentally for all nuclei here considered with the qualification that the spin is not yet known for ^{84}Nb —in the tables, three possibilities are listed: $(1^+, 2^+, 3^+)$. To form a cross-conjugate nucleus, we replace a proton by a neutron-hole and a neutron by a proton-hole. Thus one proton becomes $2j$ neutrons and three neutrons becomes $(2j - 2)$ protons. The value of Ω is then $j + (j - 1) = (2j - 1)$, and this is also $(J_{\max} + 1)/2$.

To complete the argument, we note that in the single- j -shell model a nucleus and its cross-conjugate partner should have identical spectra. This is not the case experimentally. The lighter members have $J = 2$ ground states and the heavier ones $J = (2j - 1)$ ground states. As far as the isomerism rule is concerned, we would argue that for the lighter members of the cross-conjugate pairs the shell

effects are present, which, although not strong enough to maintain identical spectra with their partners, are nevertheless strong enough to keep the $(J_{\max} + 1)/2$ states sufficiently low as to be isomeric in the lighter members and the $J = 2^+$ states to be isomeric in the heavier ones.

VII. ISOBARIC ANALOG STATES— $f_{7/2}$ VS. $g_{9/2}$

The $J = 0^+$ states in Tables II and III have isospins $T = 2$ while the other states have $T = 1$. The $J = 0^+$ states in ^{96}Ag are isobaric analog states of $J = 0^+$ states of the four proton-hole nucleus ^{96}Pd . Note that with the interactions that we have used, the $J = 0^+$ states lie much lower in the $g_{9/2}$ shell than in the $f_{7/2}$ shell, as far as a system of three protons and one neutron is concerned. There actually are two $T = 2$, $J = 0^+$ states for $(g_{9/2})^4$, only one for $f_{7/2}$. With INTd the lowest $J = 0^+$ state is at an excitation of 0.900 MeV, a prediction for ^{96}Ag . In ^{44}Sc and ^{52}Mn the excitation energies are 3.047 and 2.774 MeV respectively. Some caution must be used because of the uncertainty of the $T = 0$ two-body matrix elements in the $g_{9/2}$ shell.

VIII. A BRIEF DISCUSSION OF HIGH-SPIN STATES IN ^{96}Cd

Three very closely timed publications have appeared on the subject of isomerism for $A = 96$. In reference [11] Nara Singh *et al.* first found a $J = 16^+$ isomeric state in ^{96}Cd . Indeed at the time of this writing this is the only known state in this nucleus. A recent work by A.D. Becerril *et al.* [17] is very relevant to the work discussed here. They find two isomeric states in ^{96}Ag . They do not assign spins but they are probably 15^+ and 13^- . Then there is the work of P. Boutachkov *et al.* [18] which follows from the findings of reference [11]. They observe the direct decay of the isomeric 16^+ state of ^{96}Cd to the 15^+ isomeric state in ^{96}Ag and are able to determine the spins of this and other isomers.

Our single- j -shell calculation also yields a $J = 16^+$ isomer for ^{96}Cd (see Table IV). We see that the $J = 16^+$ state is calculated to be lower than $J = 15^+$ or 14^+ for both interactions. This guarantees isomerism in this model space. In principle this could be upset by the appearance of negative parity states and electric dipole transitions but this does not seem to be the case experimentally.

IX. SYMMETRIES IN THE $g_{9/2}$ SHELL

One good reason to study the single- j -shell limit is that certain symmetries appear which can serve as focal points as we go to larger spaces. For example, it was noted in reference [10] that in the case of four identical nucleons in

Table IV: Calculated energies of states for ^{96}Cd from $J = 10^+$ to 16^+ .

J^π	$E(\text{MeV})$	
	INTc	INTd
10^+	4.570	4.617
11^+	5.312	5.564
12^+	5.232	5.630
13^+	5.696	5.895
14^+	5.430	5.030
15^+	6.625	5.564
16^+	5.506	4.937

the $g_{9/2}$ shell with total angular momentum $J = 4$ there is a special eigenstate which emerges no matter what interaction is used. This is a seniority $v = 4$ state that does not mix with the $v = 2$ state and more surprisingly does not mix with the other $v = 4$ state (three $J = 4$ states in all). The analog of this state in ^{96}Pd appears in the spectra of ^{96}Ag and ^{96}Cd . Usually the components of the wave function appear the same no matter what interaction is used. The exception is if there are degeneracies. It was already pointed out in reference [10] that with a pairing interaction the two $v = 4$ states are degenerate and so any linear combinations can emerge from a matrix diagonalization—therefore the uniqueness of the special $v = 4$ state is lost. Too much symmetry is a bad thing. In ^{96}Cd another strange behaviour arises. With a $Q \cdot Q$ interaction there is a degeneracy of this $T = 2$ special state with a $T = 0$ state. One then encounters a technical problem: the computer program puts forth some arbitrary linear combination of these two states. One can get rid of this problem by adding a $t(1) \cdot t(2)$ interaction to $Q \cdot Q$. This will separate the two states and the $T = 2$ unique wave function will look the same as for any other interaction.

Again in ^{96}Cd it was pointed out in reference [19] that if one used a two-body interaction for which the two-body matrix elements with isospin $T = 0$ (i.e. the odd- J ones) were set to zero or to a constant, then the $J = 16^+$ state would be degenerate with a $J = 14^+$ state both having the configuration $[J_p = 8, J_n = 8]$. The wave functions and energies are shown for INTd and INTd with $T = 0$ matrix elements set equal to zero in Tables V and VI respectively. With the latter interaction, we see that for $J^\pi = 14^+$ the states have good dual quantum numbers $[J_p, J_n]$. With the INTd interaction the $J = 16^+$ state is at 4.937 MeV. There are three $J = 14^+$ states, one at 5.110 MeV, a second with mostly an $[8, 8]$ component at 6.498 MeV—these have isospin $T = 0^-$, and a $T = 1$ state at 6.703 MeV. In other words, the $J^\pi = 14^+$ state at 6.498 MeV is a pure $[8, 8]$ configuration and the others are $[6, 8] \pm [8, 6]$. For the full INTd interaction, the wave functions are more complicated. Note also that in Table VI the $J^\pi = 14^+$ state with configuration $[8, 8]$ is degenerate with the $J^\pi = 16^+$ state, also with $[8, 8]$ configuration; both states have isospin $T = 0$. We see

that the $T = 0$ two-body interaction is responsible for removing the 14–16 degeneracy for [8, 8].

Note further in Table 6 a three fold degeneracy. States with angular momenta 11, 13 and 14 all at 5.3798 MeV. Further more these states have the same dual quantum numbers (6, 8) and (8, 6). We have here a partial dynamical symmetry. The $T=0$ states that belong to this symmetry have total angular momenta that cannot occur for systems of identical nucleons i.e. they cannot occur in the single j shell in ^{96}Pd , a system of 4 neutron holes. As a counterpoint we include the wave functions for $J=12$. There is no symmetry here—the wave functions have components with various values of (J_p, J_n) . The symmetry displays itself with states with $J=11, 13, 14$, and 16—these cannot occur for 4 identical particles. Now $J=15$ also does not occur for 4 identical particles. However this state has isospin $T=1$. The symmetry only occurs for $T=0$ states.

Table V: Wave functions and energies (in MeV, at the top) of selected states of ^{96}Cd calculated with the INTd interaction.

$J = 11$						
J_p	J_n	5.5640	5.6482	6.4693	6.6384	6.9319
				$T = 1$	$T = 1$	$T = 1$
4	8	0.4709	-0.6359	-0.2463	0.3092	-0.4544
6	6	0.2229	0.0000	0.8712	0.0000	-0.3121
6	8	0.4607	-0.3092	-0.0631	-0.6359	0.4432
8	4	0.4709	0.6359	-0.2463	-0.3092	-0.4544
8	6	0.4607	0.3092	-0.0631	0.6359	0.4432
8	8	0.2869	0.0000	0.3343	0.0000	0.3110
$J = 12$						
J_p	J_n	5.0303	5.8274	6.1835	6.7289	6.8648
				$T = 1$	$T = 1$	$T = 2$
4	8	0.4364	0.3052	-0.3894	-0.3592	-0.5903
6	6	0.7797	-0.4079	0.0000	0.2927	0.0000
6	8	0.0344	0.5602	-0.5903	0.2078	0.3894
8	4	0.4364	0.3052	0.3894	-0.3592	0.5903
8	6	0.0344	0.5602	0.5903	0.2078	-0.3894
8	8	0.0940	0.1402	0.0000	0.7550	0.0000
$J = 13$						
J_p	J_n	5.8951	6.1898	7.5023		
			$T = 1$	$T = 1$		
6	8	0.7071	0.6097	-0.3581		
8	6	-0.7071	0.6097	-0.3581		
8	8	0.0000	0.5065	0.8623		
$J = 14$						
J_p	J_n	5.1098	6.4980	6.7036		
				$T = 1$		
6	8	0.6943	-0.1339	-0.7071		
8	6	0.6943	-0.1339	0.7071		
8	8	0.1894	0.9819	0.0000		
$J = 15$						
J_p	J_n	6.2789				
		$T = 1$				
8	8	1.0000				
$J = 16$						
J_p	J_n	4.9371				
		$T = 1$				
8	8	1.0000				

Table VI: Wave functions and energies (in MeV, at the top) of selected states of ^{96}Cd calculated with the interaction INTd with $T = 0$ matrix elements set to zero.

$J = 11$						
J_p	J_n	5.0829	5.3798	6.8295	7.4699	7.5178
				$T = 1$	$T = 1$	$T = 1$
4	8	0.7071	0.0000	0.2933	-0.5491	0.3351
6	6	0.0000	0.0000	0.2913	0.5605	0.6482
6	8	0.0000	0.7071	0.5350	0.0396	-0.4111
8	4	-0.7071	0.0000	0.2933	-0.5491	0.3351
8	6	0.0000	-0.7071	0.5350	0.0396	-0.4111
8	8	0.0000	0.0000	0.4130	0.2822	0.1319
$J = 12$						
J_p	J_n	5.1165	5.2336	5.4865	7.5293	7.5959
					$T = 1$	$T = 1$
4	8	0.5699	0.2803	-0.0961	-0.4783	0.5208
6	6	0.5712	-0.7151	0.1498	0.0000	0.0000
6	8	0.0925	0.3679	0.4629	-0.5208	-0.4783
8	4	0.5699	0.2803	-0.0961	0.4783	-0.5208
8	6	0.0925	0.3679	0.4629	0.5208	0.4783
8	8	-0.0846	-0.2465	0.7284	0.0000	0.0000
$J = 13$						
J_p	J_n	5.3798	7.6143	7.8873		
			$T = 1$	$T = 1$		
6	8	0.7071	0.5265	-0.4721		
8	6	-0.7071	0.5265	-0.4721		
8	8	0.0000	0.6676	0.7445		
$J = 14$						
J_p	J_n	5.3798	5.6007	7.8515		
				$T = 1$		
6	8	0.7071	0.0000	-0.7071		
8	6	0.7071	0.0000	0.7071		
8	8	0.0000	1.0000	0.0000		
$J = 15$						
J_p	J_n	7.9251				
		$T = 1$				
8	8	1.0000				
$J = 16$						
J_p	J_n	5.6007				
		$T = 1$				
8	8	1.0000				

X. ADDED COMMENTS

We emphasize here that we are making only qualitative statements about isomerism, i.e. which angular momenta are and are not isomeric. We make comparisons of cross-conjugate pairs. Cross-conjugation is a single- j -shell concept and so we invoke this model for insight into the behaviour of these four-nucleon $T = 1$ systems. We then note that memory of the single j shell persists even in larger space calculations and indeed in nature. This explains the criss-cross behaviour so that $J = 2^+$ states in lighter members of a cross-conjugate pair are ground states and in the heavier members they are sufficiently low lying so as to be isomeric. Likewise $(J_{\max} + 1)/2$ states are sufficiently low lying so as to be isomeric for lighter members and ground states for heavier members. An important point in obtaining these results is that one

should use as input the two-particle spectrum for the lighter member of the cross-conjugate pair and the two-hole spectrum for the heavier pair. The most obvious difference is that the energy of the two-hole state with $J = J_{\max} = 2j$ is much lower than the corresponding energy for two particles.

It should be further noted that the energy levels come out fairly well in the single- j -shell model when compared with experiment (see Tables I, II, and III). Note that the sudden drop in the $J^\pi = 9^+$ energy of ^{96}Ag is correctly reproduced. This shows that the single- j -shell model has considerable validity for the cases considered.

Most importantly we feel that after addressing the properties of a given nucleus, either theoretically or experimentally, one should try to see if the specific results are part of a bigger picture. This is certainly the case here. For example, the striking analogous behaviours in ^{52}Mn and ^{96}Ag lead us to conclude that both $J = 2^+$ and $(J_{\max} + 1)/2$ states should be long-lived.

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Appendix A: Interactions discussed in this work

We first show in Table VII the two-body matrix elements that we used in this work in increasing spin from $J = 0$ to $J = J_{\max}$. The even spins have isospin $T = 1$ and the odd ones $T = 0$.

We next consider the large scale interactions. In Ref. [6] the KB3 interaction was used in a complete f - p space ($f_{7/2}, p_{3/2}, f_{5/2}, p_{1/2}$) for the study of ^{52}Fe ; in Ref. [7] the GXPF1A interaction was used for the same nucleus and model space. In Ref. [13], to study mainly ^{96}Pd , the SLG and F-FIT interactions were used in the model space ($p_{1/2}, g_{9/2}$), together with the JS interaction in a somewhat larger model space (allowing single-nucleon excitations to the orbitals $g_{7/2}, d_{5/2}, s_{1/2}, d_{3/2}$). Again the model space ($p_{1/2}, g_{9/2}$) was used in Ref. [17] for ^{96}Ag with the SLGT interaction, while the jj44b interaction was also used but within the model space ($p_{3/2}, f_{5/2}, p_{1/2}, g_{9/2}$). Finally, in Ref. [18] various interactions were used: GF in the space ($p_{1/2}, g_{9/2}$), FPG in ($p_{3/2}, f_{5/2}, p_{1/2}, g_{9/2}$), and GDS in ($g_{9/2}, g_{7/2}, d_{5/2}, s_{1/2}, d_{3/2}$).

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Table VII: Two-body matrix elements in increasing spin from $J = 0$ to $J = J_{\max}$. The even spins have isospin $T = 1$ and the odd ones $T = 0$.

	$f_{7/2}$		$g_{9/2}$		$h_{11/2}$
J	INTa	INTb	INTc	INTd	$Q \cdot Q$
0	0.0000	0.0000	0.0000	0.0000	-1.0000
1	0.6111	0.5723	1.1387	1.1387	-0.9161
2	1.5863	1.4465	1.3947	1.3947	-0.7544
3	1.4904	1.8224	1.8230	1.8230	-0.5325
4	2.8153	2.6450	2.0823	2.0823	-0.2687
5	1.5101	2.1490	1.9215	1.9215	0.0070
6	3.2420	2.9600	2.2802	2.2802	0.2587
7	0.6163	0.1990	1.8797	1.8797	0.4434
8			2.4275	2.4275	0.5105
9			1.4964	0.7500	0.4026
10					0.0549
11					-1.6044

(2011).